A COMPANION to The LADIES and GENTLEMENS DIARY, FOR THE YEAR 1779:

CONTAINING

ÆNIGMAS, REBUSSES, MATHEMATICAL ESSAYS, QUESTIONS AND SOLUTIONS, &c.

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Answers *to the* Queries, Rebusses, *&c. in Last Year's* Diary.

Query I. *Answered by Mr.* Dalby.

THE *calf* of the leg seems to have had its name from two *cimbric* words, *cal*, stout (or large, with respect to the other parts of the leg) and *lef*, always bent, or of a bended form [*vid. Goropius Becanus*] and from thence the Dutch name *kalf*, from this the English is evidently borrowed.

Query II. *Answered by Miss* Greville.

When a piece of iron is heated red hot and cooled in the open air its bulk becomes greater, or, it occupies more space, and therefore the particles composing it are at a greater distance from each other than before, and consequently the whole is less compact and softer; but the contrary happens if cooled in water; for, in heating, a great part of the air it contained is excluded by that operation on account of its expansion and rarefaction; then suddenly plunging it in water, the air is thereby prevented from insinuating itself into the metal while it cools, and so the particles, having more room, fall nearer together, which evidently must render it of a firmer texture.

Query III. *Answered by Dr.* Slop.

As Cotton only meant by the words in question to give a burlesque representation of the violence of the storm; his intention was evidently to compare its effects on the world with those of wine on the head of a drunkard; and as the famous Barnaby Harrington was not long before Cotton's time, so remarkable for his drunkenness and his poetry, he is doubtless the person alluded to; and therefore to *dance Barnaby*, is only another expression for *reeling*.

Query IV. *Answered by Miss* Polly Lee.

Problems in plane geometry can be drawn more exact with great distances than with small, because all points and lines in *practice* are of some breadth, and such breadths will hold a less proportion with great than with small distances, and consequently the errors in drawing will be less in using long lines than short ones: To explain this, suppose the circumference of a circle whose diameter is one-tenth of an inch, is to be divided into 1536 equal parts by lines drawn from the centre, this we will suppose to be done by a continual bisection of the chords, now when we come to the last divisions, we shall find that the lines which are to divide the chords, will be as broad as the chords are long, though perhaps the instrument may be as fine as possible; but this would not be the case if the diameter was two or three yards.

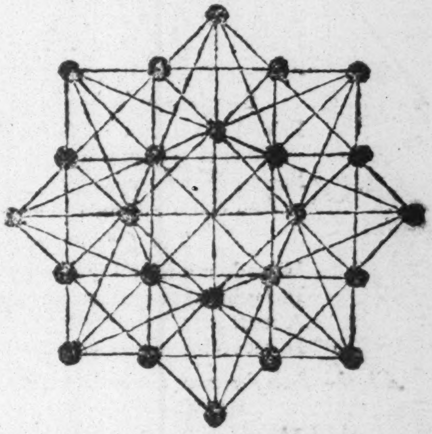
Query V. *Answered by* Caput Mortuum.

Whence the salt of the sea proceeds is a curious but difficult problem. According to some Naturalists it is owing to the mines of *Sal Gem* in the bowels of the earth washed down by the rains;-- admitting this the Sea must grow continually salter, because the water raised by evaporatio

poration is freſh. On this ſuppoſition Dr. *Halley* propoſed a theory to determine how long the world has ſubſiſted; but as obſervations have not been made of the degree of its ſaltneſs at diſtant periods of time, it muſt be left to the determination of the curious in future ages.

Anſwer to the Paradox, by Mr. John Wales.

Behold, honeſt *David*, your leeks I have planted
In twenty-eight rows, and exact as you wanted;
In the margin pray view them, nor take them on truſt,
You may count all the rows, for the number is juſt.



In this manner it was Anſwered by Mr. Silcock, *and many others.*

Rebus II. *Anſwered by Mr.* John Clarke *of* Lincoln.

Dalby, we know thou canſt with eaſe write well,
But joke no more about that place call'd *Hell.*

Anſwer to the Acrostic Rebus, *by* James Twitcher.

Perfidious France and haughty Spain
Are at their *dirty tricks* again;---
Had but King George adviſ'd with *Chatham,*
The Britiſh Lion had been at 'em.

All the Rebusses *anſwered by Mr.* John Wales.

I find 'tis fam'd *Newton* Miſs Lee ſtrives to hide, I.
And *Hell* is the place where the wicked reſide, II.
Miſs Greville to *nobody* ſeems to be join'd, III.
But Beatrice the fair is in *marriage* combin'd, IV.
Great *Milton* and *Chatham* then bring up the rear; V. VI.
So I hope all the Rebuſſes I've made appear.

Answers *to the* Enigmas *in laſt Year's* Diary.

I. *A maidenhead*	VII. *A key*
II. *Nothing*	VIII. *A bum* or *deceit*
III. *A mouſetrap*	IX. *Patience*
IV. *Tobacco*	X. *Nonſenſe*
V. *Charity*	Prize. *A kiſs.*
VI. *Yourſelf*	

The

The Prize Enigma answered by Mr. Joseph James.

Let poets talk of that or this,
L. Walker well describes a *kiss*.

The same answered by Cymon.

As Chloe was sleeping, Hodge view'd the dear maid
With a rapt'rous eye, and exultingly said;
I'll snatch a sweet *kiss*, ere she opens her eyes,
And claim both the *gloves* and the Diary Prize!

The same answered by Milito *of Thingdon*.

Friend Walker, a *kiss* is sure highest bliss
To each true and lovely sweet pair,
But oft the fair maid by kissing's betray'd,
And artfully drawn in a snare.

The same answered by Silvia.

Indeed, my friend *Walker*, you're perfectly right
In saying that *kissing* affords great delight,
A most pleasing sensation fills every vein
In receiving *salutes* from a favorite swain.

All the Rebusses and Enigmas answered by Miss Eliz. Cockbill, *of Mansfield Woodhouse, in Nottinghamshire.*

My *maidenhead's* my *hope*, my pride, I. IX.
I've beauty's *mousetrap* on my side; III.
The flatterer's *key*-like *kiss* I'll shun, VII. *Prize*.
And from the *smoaking* sot I'll run; IV.
Of prating fops I'll take no heed,
Their *nonsense nothing* can exceed; X. II.
Nor shall *deceit* my actions stain; VIII.
Nor *self* conceit over me reign; VI.
But I'll be jovial, gay and free,
And live with all in *charity*: V.
But, if my lucky stars design
That I in *marriage* knot shall join, IV. *Reb*.
Then may my consort be as rare
As *Milton* or great *Newton* were. V. I. *Reb*.
With him then I'll to *Chatham* sail, VI. *Reb*.
And there steer thro' life's chequer'd vale.
I'll fear *Nobody's* envious spell, III. *Reb*.
Nor all the serene wights of *Hell*; II. *Reb*.
But to my love I will prove true
'Till Death commands to bid adieu.

General Answer to the Enigmas, by Mr. Leonard Walker.

Accept, my dear Lucy, advice from a friend,
And adhere to the rules which I now recommend:
Tho' Strephon adores you, be careful of this,
To repulse his bold freedoms nor grant him a *kiss*, *Prize*.
Such freedoms admitted will lead him to more,
'Till he gains the *last favor* and makes you a w---e! I.

Let

Let *nothing* perſuade you, not love, praiſe, or pelf, II.
To forget for a moment what's due to *yourſelf*. VI.
Let *meekneſs* and *charity* guide you thro' life, V. IX.
They're the *keys* of true knowledge for maid or for wife; VII.
And will lead you the *nonſenſe* of fops to deſpiſe, X.
And to laugh at the pert and affectedly wiſe;
No *deceit* ever uſe to *entrap* the unwary, VIII. III.
'Twill end but in *fumo* and ſurely miſcarry. IV.
This advice, my dear Lucy, is homely and plain;
Yet I think it can do you no harm in the main.

General Anſwer to all the Enigmas, by Mr. John Clarke *of Lincoln.*

As lately I my annual journey went,
At *Maidenhead* a pleaſant evening ſpent: I.
Nothing was wanting to increaſe our mirth, II.
And drinking deep gave noiſe and *nonſenſe* birth; X.
Snug as are mice within their *wiry-houſes*, III.
We drank remembrance to our abſent ſpouſes;
Smok'd *Trinidado* whilſt we drank our nappy, IV.
Sung glees and catches and were vaſtly happy.
No narrow minded wretch diſgrac'd our board,
Nor ſuch who cannot *charity* afford; V.
But all were ſocial, jovial, merry ſouls,
And briſkly puſh'd about the flowing bowls;
My faculties grew weak, the liquor ſtronger,
I ſoon was tipſy and could ſtay no longer,
Then ſlily left them as my fancy led,
And took *myſelf*, with decency to bed. VI.
I *lock'd* my door that none ſhould therein come, VII.
And laugh'd that I had play'd them ſuch a *bam*. VIII.
With ſweet *content* I ſlept away the night, IX.
And did not wake until the morning light.
The woodlark's ſong proclaim'd th' approaching morn,
And Phœbus rays *ſalute* the waving corn. *Prize.*

Ingenious Anſwers to the Queries, Rebuſſes, Enigmas, &c. have been received from Miſs Lee, Miſs Brown, Silvia, Beatrice, and Meſſ. Rogers, James, Pepys, &c. which for want of room we are obliged to omit. The Prize of ten Diaries fell to the lot of Miſs Eliz. Cockbill; who is deſired to ſend to Mr. Carnan's, N°. 65, St. Paul's Church-Yard, for them.

New Queries, Rebusses, *&c. to be anſwered in next Year's Diary.*

I. Query, *by Mr.* Dalby.

How are we to underſtand this expreſſion of Job, Chap. xxvi. ver. 5. Dead things are formed from under the waters, and the inhabitants thereof?

II. Query, *by* [illegible]. John Clarke.

Upon what natural principles are we to account for a ſeemingly total alteration of *air*, *climate*, and *ſeaſons*, in many different parts of the

the world? Why are *summers* in *England* become of late years so wet and winterly, and in general, the weather so uncertain and variable? And why is *Montpelier* not the salutary spot it used to be?

III. Query, *by Mr.* John Wales.

Diarian artists, make appear
How long since *hats* first came in wear.

IV. Query, *by Mr.* Thomas Hoy.

In the second chapter of Samuel, it is mentioned that Eli's sons used a flesh hook to pull meat out of the pot. *Query*, was this *hook* barbed or not?

V. Query, *by Mr.* J. Burrow.

If a person breathes upon the blade of a new knife, razor, &c. the moisture immediately flies off. What is the reason of this?

VI. Query, *by Miss* Polly Lee.

As candles, &c. burn much faster in Dr. Priestley's, than in common air, might not some useful method of introducing and confining a similar kind of air in the substance of gunpowder, be contrived, in order to render it more forcible and instantaneous in its explosion?

I. Rebus, *by Miss* Dale.

A quarter of what at the tavern you spend,
And what's without either beginning or end,
With three fifths of a broad grin,---connected together,
Is a teazing companion in hot or cold weather.

II. Rebus, *by Mr.* Dalby.

The reverse of a hue that young Phillis can boast,
And three fifths of a knife when the handle is lost,
If join'd,---to the knowledge of something you're led
That always grows thinner the more it is fed.

III. Rebus, *by* Aylesbury Jack.

Two thirds of a *fib* and two thirds of *hard-water*,
Are the foes which French soldiers pursue with great slaughter.

IV. Rebus, *by* Caput Mortuum.

Half a noun with as much of what's commonly hollow,
Is a runner, and one you're obliged to follow.

V. *Paradoxical* Rebus, *by Mr.* G. Pepys.

When to just half a thousand one evil you stick,
You've a damnable fellow as fierce as Old Nick.

VI. Rebus, *by Mr.* L. Walker.

A Roman chief, who Sylla's pow'r defy'd;
A Grecian hero, who by Paris dy'd;
A famous hunter, who a city built;
A man, who's charg'd in orient climes, with guilt;
An era, which the Turkish people prize;
A bird, which views the sun with steady eyes;

A sage

A sage of Athens, who their laws revis'd;
A Roman prince, by all mankind despis'd;
A man, whose chief companion was his lamp;
What should be always near a soldier's camp.
Th' initials join'd, a town will bring to view,
For trade and riches parallel'd by few.

A Paradoxical Problem, by Mr. L. Silcock.

Diarian artists, if you please
To plant me thirty cherry trees,
Thirty-four rows, nor less nor more,
And in each row exactly four.

New ENIGMAS *to be answered in the next Year's* DIARY.

I. ENIGMA, *by Miss* Kitty C——.

I from the earliest ages date my birth,
Yet am not seen in water, air, or earth,
Fond of retreat, I seek the shady grove,
A foe to friendship, but a friend to love;
The powers of music have no charms for me,
Yet strange to tell, I'm fond of harmony,
And tho' with wisdom I am known to dwell,
And calm content admits me to her cell;
Yet blushing, ladies, I my weakness own,
To virtue I am utterly unknown.

II. ENIGMA, *by Miss* Dale.

Tell me, ye learned fair ones, what is this
Which all admire, yet very few possess;
A virtue 'tis, to ancient maids unknown,
And prudes who spy all faults, except their own:
Lov'd and defended by the brave and wise,
Tho' knaves abuse it, and like fools despise.
Secure of me you can no envy move,
For none can envy those whom all must love.
In fact, my power adds a brighter grace,
And sweetens every charm in Sylvia's face.

III. ÆNIGMA, *by the late* Thomas Sadler.

Swift as the wind I cleave the liquid air,
When to my destin'd goal I would repair:
Oft doth the flying deer my fury own,
And bravest warriors hail me with a groan.
Sometimes in sportive mood by Perseus' arm
I have been sent to strike a dire alarm,
Amidst the feather'd race; who soaring high,
By me arrested quit their native sky,
Rapid they fall in circling eddies round,
And strike their talons in the senseless ground.

Bold Robin Hood in me was greatly ſkill'd,
And often took me with him in the field!
Deſpair and horror mark'd my fearful way,
And ſtouteſt heroes ſhudder'd with diſmay.
Sometimes leſs fear'd, tho' felt with great ſurprize,
I take my ſtation in fair Chloe's eyes:
Then do the beaux my dreaded power try,
And pining lovers in a moment die.
Whene'er ſly Cupid would invade a heart,
He then invokes my never failing art;
Secure in me he ſtrikes the fatal blow,
With love's hot fire the virgins boſoms glow.
Sweet ſleep forſakes their eyes, and from their breaſt,
Are baniſh'd pleaſing thoughts, and balmy reſt.
Tell me, ye fair, my name,---from whence I come,
And may your cheeks preſerve unfading bloom.

IV. Enigma, *by Mr.* John Clarke *of Lincoln.*

Altho' I'm us'd by ev'ry one,
  Of high or low condition,
Yet ſeldom am confin'd alone
  To lawyer or phyſician.
I'm thought miſchievous as a cat,
  On various pretences;
Strange tricks and whimſies I've been at,
  When playing with the wenches.
No town within our king's dominions,
  But to my talents claim a ſhare,
Yet people form abſurd opinions,
  And curſe me oft for being there.
I'm cenſur'd by the keeneſt tongue,
  Severely loaded with abuſe,
And drag a wretched life along,
  Alas! becauſe I'm not of uſe.
Innocent I am by nature,
  Free and light as noontide air;
No harſh lineament or feature
  Ever in my face appear.
The lies of tradeſmen, politicians,
  And lottery-ticket ſellers,
Philoſophers, theologicians,
  Cock-Lane ghoſt, or Punchinellos,
The canting zeal of puritans,
  And Tabernacle preachers,
The honeſt ways of courtezans,
  And vile death-hunting-ſearchers;
All theſe the world will not believe,
  As faith is not their due,
But pin their credit on my ſleeve,
  Becauſe I think them true.

V. Enigma,

V. Enigma, by Mr. B. Cleypole, of West-Ham, Essex.

Ye sage enigmatists, make room,
And let your well known fav'rite come,
Caress'd by all the English nation,
And holds distinguish'd reputation:
But stop--perhaps you now will swear,
Like boasting Falstaff I appear;
Because when view'd by some they tell ye
Like him I'm little else but belly;
Yet I'll confess altho' esteem'd
By others I'm a monster deem'd.
No Sphinx or Hydra to surprize,
Or Argus with his hundred eyes;
Of such contexture is my frame,
A mouth and body's all I claim;
And tho' I'm neither flesh nor blood,
Like human beings crave my food;
Th' Epicure whose constant care
Is costly dainties to prepare,
Has seldom such profuse supply
Of rich and sumptuous fare as I;
But fate and knaves at once conspire
That tortur'd I must be with fire;
And circling flames my entrails burn,
Which unto smoke and ashes turn,
Whilst I, oh horrible to tell,
Am emblematical of hell;
But of those torments I make light,
Because they bring an appetite;
For glutton-like know I receive
All food that my attendants give.
Amazement doubtless 'twill create,
When I this well known tale relate;--
What I disgorge none will deny
Proud mortals eat, or else must die;
Enough is said; declare my name,
And to the world my worth proclaim.

VI. Enigma, by Mr. Clarke, of Farnham, n Surry.

Ladies, a female slave behold,
That's sore opprest by young and old,
And begs you'll shew some pity on her,
For friends all turn their backs upon her.
Nor think that I your suppliant crave
Your aid unmerited to have,
Or wish like saints of Aaron's trade
To labour less the more I'm paid;
For tho' I might exemption claim,
Because your flesh and mine's the same;

Or plead that by my ſhape's confeſt,
I ne'er was meant for work, but reſt;
Yet ſuch like Iriſh pleas ſome dozen,
I leave to Paddy's couſin's couſin,
Who'd rather than not live at caſe,
Lie down with dogs and riſe with fleas.
Tho' ſmall my limbs, yet in a year
I tons of h-ly garbage bear;
Tho' ſlender I am often put
To carry loads of learned gut;
Oft laden with this ponderous freight,
I groan beneath the ſinful weight;
Yet not from weakneſs or from fear,
For know that I rude ſhocks can bear,
Not blaſts of rattling winds can move me,
Nor thunders when they roll above me.
The higheſt nobles cringe to me,
The greateſt monarchs bend the knee,
For my aſſiſtance many ſue
In public and in private too;
Ev'n ladies take me oft in hand,
And when I fall they make me ſtand;
Nor can without my aid divine,
The lawyer, judge, or biſhop dine,
For I alone uphold them all,
And but for me the Pope would fall.
In my embrace has Sawney R——
Oft ſeen the lovely *Polly Stow*,
With Ragged Robin at her ſide,
In flower of ***and ****y pride.
Tho' I without the help of man,
More children bear than women can;
Yet think not I'm devoid of charms,
For men oft ſleep within my arms;
Nay more, my miſtreſs oft hath ſeen
Me take my maſter in between.
  Scylla, as ſing poetic rogues,
Was often lin'd with her own dogs,
When danger threaten'd,---coward rout!
In ſuch like caſes mine jump out.
  When right divine was much in vogue,
I was your non reſiſting rogue;
When high church tories were in power,
I play'd th' obedient paſſive whore;
When drunken ſcoundrels rul'd the ſtate,
I kept bad hours and ſat up late,
Ev'n in theſe virtuous ſober times,
I often join in wicked crimes;
When bloods to bawdy houſes come,
I bounce and fly about the room;

But

But when they to old Fielding trudge,
I ſtand as ſober as a judge.
In former ages I was furr'd,
I next went bare as any board,
But if you're now diſpos'd to find
My hairy bottom, look behind.

VII. ENIGMA, *by Mr.* Leonard Walker.

Ye wiſe and prudent, lend a patient ear
To one, whoſe friendſhip you ſhou'd all revere;
No needy ſharper claims your ſage regard;
To ſerve you well, ſhall be my ſole reward.
I owe my being to the fruitful earth;
An artiſt form'd and gave that being worth:
All civil ſtates my virtues highly prize;
Tho' nature's ſons my nobleſt deeds deſpiſe.
In this bleſt iſle, where juſtice freely flows,
My perſon's ſacred, by the wiſeſt laws;
The hapleſs wretch, who freedom takes with me,
Oft forfeits fame, and dearer liberty!
Whilſt impious men religion diſbelieve,
At church I ſtill attend both morn and eve;
Nor Pope, nor council, tho' their zeal runs high,
Are truer guardians to the church than I.
I'm ſtill the trav'ller's true and ſteady friend;
And in his journies I on him attend;
In foreign climes, tho' he ſhould long ſojourn,
With zeal I ſerve him, 'till his wiſh'd return.
The hoary miſer oft in me confides,
To guard that wealth, which he with caution hides;
He ſafely may; I ne'er betray my truſt;
Thrice happy Briton, were thy ſons as juſt!
The ſtage I ſometimes tread, with great ſucceſs,
All ranks applaud me, and my worth confeſs;
Tho' Roſcius was the fav'rite of the town,
He never gain'd more fair and juſt renown!
No maſter, yet, cou'd ever truly ſay,
That I demanded either thanks or pay,
Tho' watch and ward I keep, both night and day.
To heap up wealth, the world's great end and aim,
I leave to thoſe, who ſigh for pomp and fame.
Ye lovely fair! may I permiſſion crave,
To prove, that I'm the deareſt friend you have;
Your warmeſt friends muſt yield the palm to me,
For I'm the guardian of your chaſtity!
Tho' England's fair ones may my pow'r diſclaim,
Iberia owns it, and reveres my name.
But I muſt now attend my maſter's call;
Adieu!-- and may ſucceſs attend you all!

VIII. ENIGMA,

VIII. Enigma, *by Miſs* Eliz. Cockbill.

Ye gentlemen in verſe ſublime,
Excuſe a female bard's weak rhyme;
Nor think like you I couplets chuſe,
Nor in ſuch ſtrains invoke the muſe;
Nor ſhall I ſing of lawns nor rills?
Nor flow'ry vales nor lofty hills,
Nor of old Nereus nor his ſtream,
Nor take the Sylvans for my theme.
But ſuch a topic let me chuſe,
As ſportive ſouls can ne'er refuſe.
And, Sirs, if you my theme wou'd trace,
You muſt conſult old Nimrod's race;
And follow cloſe whate'er betide,
For to Actæon I'm ally'd.
Tho' I'm no man, no bird, nor whale,
I've neither ſhoulders, hair, nor tail.
Two wings I have, tho' never fly,
Nor direct objects can eſpy.
My enemies I ne'er offend,
Tho' many often ſeek my end.
My habitation or retreat,
Is a ſweet pleaſant country ſeat;
Secret's my cot, and ſeldom found
Either above or under ground:
Where thro' a life of fears I run,
And range alternate with the ſun.
Whore is my name, and long has been,
Tho' with my gallants ſeldom ſeen.
And if by chance mankind I paſs,
They term me of the female race.
Yet often in this maſk of mine
Is wrapt a ſubſtance maſculine!
'Tis ſtrange! Each ſex in me unite,
Yet ſtill I'm no hermaphrodite!
But, Sirs, if you my ſhape wou'd know,
Pray look for me when at Soho!

Prize Enigma *(of 10 Diaries) by Mr.* Dalby.

Ye *beaux*, and *feather'd belles* attend;---
At my approach obſequious bend,
Nor ſhun me, or you may expect
In tears to mourn the dire neglect!
While I, a handmaid of the graces,
Shall cauſe you many damn'd wry faces.
Is there who've ſeen, in Eaſtern pride,
The *Great Mogul* triumphant ride?---
Upon the ſelf ſame beaſt that bore him
'Tis two to one I've rid before him;---
For know, I am, ſuch is your will,
The higheſt office born to fill.---

No

No minister with all his arts
Can boast such penetrating parts.
In vain the tyrant strives to hide
From me, who am the scourge of pride:
For, should a wight, tho' high in place,
Yet, born of mean ignoble race,
Ambitiously usurp a crown,
I pull the vile pretender down;---
While on the verge of fate he reels,
Mankind perhaps my vengeance feels:
Yet, let me not increase your fear,
A meagre form at best I wear;
And, tho' you often me will find
Like two with backs together join'd,
A fool who shouldn't chance to know me
With half an eye may see quite thro' me.
When with majestic pace, you oft
In querpo see me ride aloft,
Tho' I am not accus'd of fear,
Yet, coward like, I chuse the rear;
There firmly fix'd, and safe from harms,
Am half eclips'd by *Chloe's* charms.---
When crosses vex I make a stand,
Nor do my business out of hand;
Yet, *Garrick* like,---for I'm but small,
The part I play is capital;
But oft employ'd like statesmens tools
In dirty jobs for knaves and fools,
Then like some grov'ling dunce, you see
That blockheads are my company;
This, as a reason some expound
With *monsieur* why I'm always found;
Tho' with him seldom I'm at peace,
And like him often *out of case*.
Taffy, Got bless hur, it is said
With me ne'er troubles much his head,
Yet, from his eyes at *David's* shrine
I once a year extract the brine,
Which falling from his russet cheeks
Is salt to toasted cheese and leeks:--
Nor let that image turn you sick,---
I'm the *Arcanum Cephalic*;
No *Pulvis Nostrum* equals me,
From *dirty Scotch* up to *Rappee*.
After what's said, I need not name
That I a bold intruder am;
Nay, impudent, for 'tis averr'd
I once catch'd *Moses* by the beard.---
But hold,---for now I make no doubt
E'en *Numscull's* self can find me out.

Anſwers to the Mathematical Queſtions *propoſed in laſt Year's* Diary.

I. Question, *anſwered by Mr.* Joſeph Bird, *jun. of* Ipſwich.

By dividing the ſecond equation by the firſt, $x^2-xy+y^2=\frac{b}{a}$, which taken from the firſt, gives $2xy=a-\frac{b}{a}$, or $xy=\frac{a^2-b}{2a}=d$ by adding this to the firſt equation, and ſubtracting it from the above, we have $x^2+2xy+y^2=a+d$, and $x^2-2xy+y^2=\frac{b}{a}-d$, and extracting the ſquare root, $x+y=\sqrt{a+d}$, and $x-y=\sqrt{\frac{b}{a}-d}$, whence $x=\frac{1}{2}\sqrt{a+d}+\frac{1}{2}\sqrt{\frac{b}{a}-d}$, and $y=\frac{1}{2}\sqrt{a+d}-\frac{1}{2}\sqrt{\frac{b}{a}-}$

In the ſame manner it was anſwered by Mr. Hugh Weetman *of* Bennington; *Mr.* W. Watſon *of* Alnwick; *Meſſrs.* James, Merritt, Barker, John Clarke *of* Lincoln, *and* Hatton *the propoſer*.

II. Question, *anſwered by Mr.* Joſeph James *of* Stoke-Biſhop, *near* Briſtol.

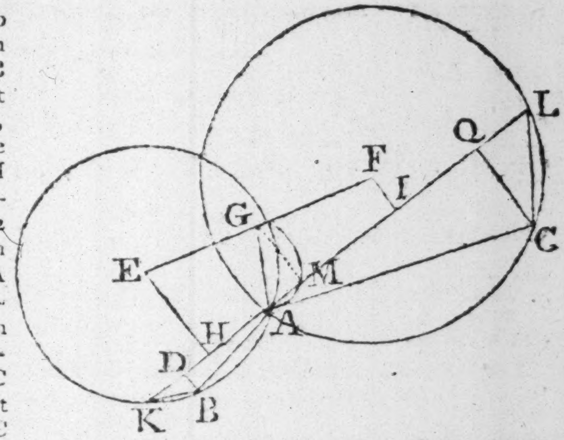
Let x and y be the required numbers; then x^2+y^2-1 and x^2-y^2-1 are ſquares, and their difference is $2y^2$, which being reſolved into the factors $2y$ and y, and half the difference ſquared and made equal to x^2-y^2-1, gives $x^2=\frac{5}{4}y^2+1$, which aſſume equal to the ſquare of $1+vy$ and y will be found equal $\frac{8v}{5-4v^2}$ and conſequently x equal $\frac{5+4v^2}{5-4v^2}$, in which if v be taken 1, $y=8$ and $x=9$.

Nearly in the ſame manner this queſtion was anſwered by Meſſrs. Ainſworth, Barker, Clarke, Fininley, Hedley, Moody, Merrit, Smith, Swift, Watſon, Weetman, *and the propoſer*.

III. Question, *anſwered by Mr.* Ainſworth.

Upon AB, AC including the given $\angle$ BAC, reſpectively deſcribe ſegments of circles, containing $\frac{1}{2}$ right $\angle$'s. And biſect EF, the line joining their centers in G, draw GA, and upon it deſcribe the ſemicircle G M A, in which inſcribe the chord A M, equal to $\frac{1}{4}$ the difference between A B and A C. Then upon A M produced, let fall the $\perp$'s B D, C Q, and the thing will be done.

Demon.



Demonst. For produce A D, A Q to meet the circles in K, L. and draw B K, C L. Also let fall the ⊥'s E H, F I. Then because E G = G F, H M = M I. consequently A I — A H = 2 A M = ½ the given difference. And A L — A K = 2 A I — 2 A H = the given difference = A C — A B, or A L — A C = A K — A B. But because the ∠'s C L Q and B K D are by construction ½ right ∠'s, Q L = Q C and D K = K B. Therefore A L = A Q + Q C, and A K = A D + D B, and consequently A Q + Q C — A C = A D + D B — A B, or which is the same thing, the diameters of the circles inscribed in the △'s A Q C and B D A are equal. — Q. E. D.

Algebraical solutions were also received from Mr. Hardy, *the proposer, and several others.*

IV. Question, *answered by Mr.* William Watson *of* Alnwick.

From the given equation is had $\dot{y} = \frac{a\dot{x} + x\dot{x}}{\sqrt{a^2 - x^2}}$, or by taking the fluents $y =$ arch whose sine is x to rad. $a. - \sqrt{a^2 - x^2}$. Also $x\dot{y} = \frac{ax\dot{x} + x^2\dot{x}}{\sqrt{a^2 - x^2}}$, whose fluent is $\frac{1}{2} a \times$ arch, whose sine is x to rad. $a - \overline{a + \frac{1}{2}x} \times \sqrt{a^2 - x^2}$. Or by supposing that $x = o$, when $y = o$, the correct fluents of $\dot{y}$ and $x\dot{y}$ will be $a - \sqrt{a^2 - x^2}$ + arch, whose sine is x to rad. a. And $a^2 + \frac{1}{2} a \times$ arch, whose sine is x to rad. $a - \overline{a + \frac{1}{2}x} \times \sqrt{a^2 - x^2}$, which last taken from $xy = ax - x\sqrt{a^2 - x^2}$ + $x \times$ arch, whose sine is x to rad. a — will give the fluent of $y\dot{x}$.

In the same manner it was also answered by Mr. Ainsworth; *and the answers given by Messrs.* Bonnycastle, Barker, Caput Mortuum, Hampshire, James, Merritt, Pepys, Weetman, *and the proposer, were nearly similar.*

V. Question, *answered by Mr.* Ainsworth, *and the proposer.*

Put $\frac{1}{1.05} = r$. and for convenience suppose $a = 1$. Then the present value of the whole estate will be $r + 2r^2 + 3r^3 + 4r^4 +$ &c. =

$=\frac{r}{\overline{1-r}|^{2}}$ . . And the value of the reversion, or the terms of this series, after the first x are $\overline{x+1.r}^{\,x+1}+\overline{x+2.r}^{\,x+2}+\overline{x+3.r}^{\,x+3}$ + &c. $=xr^{x}\times\overline{r+r^{2}+r^{3}+r^{4}+\&c.}+r^{x}\times\overline{r+2r^{2}+3r^{3}+4r^{4}+\&c.}=xr^{x}\times\frac{r}{1+r}+r^{x}\times\frac{r}{\overline{1-r}|^{2}}$, which must per question be $=\frac{1}{2}\times\frac{r}{\overline{1-r}|^{2}}$, that is $xr^{x}+r^{x}\times\frac{1}{1-r}=\frac{1}{2}\times\frac{1}{1-r}$. In numbers $xr^{x}+21r^{x}=10\frac{1}{2}$, or $r^{x}\times\overline{x+21}=10\frac{1}{2}$, from which x may easily be found $=33,9$, or nearly 34 years, the time required.

The answers given by Messrs. Bonnycastle, Dalby, Hampshire, James, Sanderson, *and* Pepys, *were nearly the same as above.*

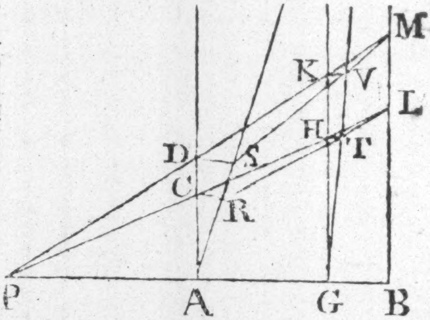
N. B. The sixth and seventh questions were inserted by mistake, the one having already been answered in Mr. *Lawson*'s Tangencies, and the solution of the other taking up more room than is consistent with the limits of the present diary.

VIII. Question, *answered by Mr.* Ainsworth.

Let s be the required sum, and $2n+3=m$, or $m=2$, then by the nature of the series $\overline{m-2}\times\overline{m-1}|^{2}\times m^{3}=s$, or $m^{4}-4m^{3}+4m^{2}-2m=s$ The correct integral of which is found by page 46, Emerson's Increments, to be $s=\frac{m^{7}}{14}-\frac{5}{6}m^{6}+\frac{7}{2}m^{5}-6\frac{1}{12}m^{4}+3m^{3}+\frac{5}{3}m^{2}-\frac{4}{7}m-\frac{3}{4}$.

This question was also answered by Mr. Ur. Bowerburne, *from* Sterling's *Differential Method, and by Mr.* George Sanderson, *and Mr.* John Bonnycastle *according to the method of Increments.*

IX. Question, *answered by Mr.* John Hampshire.



Let B A be produced to P so that B P may be to P A in the given ratio of B L to A R and B M to A S and draw A C and G H parallel to B M, and join P L, P M meeting A C and G K in the points C, D, H, K and join the points C, R and D S, H T and K V.

Then by construction, B P : P A : : B L : A R but B P : P A : : B L : A C therefore A C = A R and by the same reason A D = A S and D S parallel to C R; again, because A D and G K are parallel, B G : G A : : L H : H C : : M K : K D, but B G : G A : : L T : T R : : M V : V S

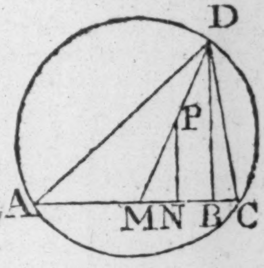
V S by hypothesis; wherefore C R, D S, H T and K V are parallel; now H T : C R : : B G : B A and C R : D S : : A C : A D, and D S : K V : : (M D : M K) : : B A : B G, therefore H T : K V : : A C : A D but A C : A D : : G H : G K therefore G H : G K : : H T : K V and consequently the points G, T, V which divide the line. B A, L R, M S in the given ratio of B G to G A are in a right lines

Scholium. This question is in effect the same as Newton's 23 *lemma, which was not observed by the proposer at the time of sending it; however it will doubtless be agreeable to the readers to have different solutions to so useful a problem.*

In the same manner it was answered by Mr. Ur. Bowerburne; *very elegant geometrical demonstrations were also received from Messrs.* Ainsworth, Sanderson, *and* Moss *the proposer.*

X. Question, *answered by Mr.* William Fininley.

Construc. Bisect the base A C in M, on A C describe a segment containing the given vertical angle, and take any line M N and N P perpendicular to it, so that M N may be to N P as the radius of the circle to the given base; join M P meeting the circumference in D, then A D C is the triangle required.



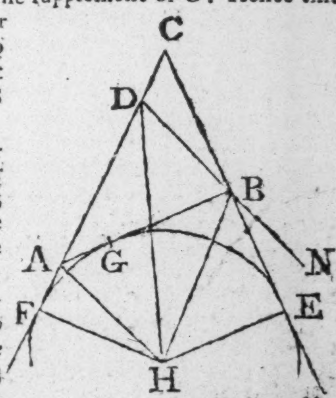
For draw D B parallel to N P then M B : B D : : R (radius) : A C therefore D B × R = A C × M B consequently D B × diameter $=$ A C × 2 M B $= AD^2 - DC^2$; but D B × 2 R = A D × D C by 6. Eucl. therefore A D × D C $= AD^2 - DC^2$.

Geometrical constructions were received from Messrs. Ainsworth, Barker, Moody, *and* John Clarke *of* Lincoln; *and very elegant algebraical solutions from Messrs.* Watson, James, Weetman, Mertitt, Pepys *and* Caput Mortuum.

XI. Question, *answered by Mr.* George Sanderson.

Analysis. Let C D be the given difference, then because A D and A B are equal, A D B = A B D, and F A H being half F A B is equal to A D B, therefore A H and D B are parallel, consequently A H B = H B N but A H B is half the supplement of A C B (by Prop. 3, Diary 1777) therefore H B N is half the supplement of C: Hence this

Construc. On H D describe a circular segment containing an angle equal to half the angle C together with a right angle, cutting C E in B, then A C B is the triangle required.



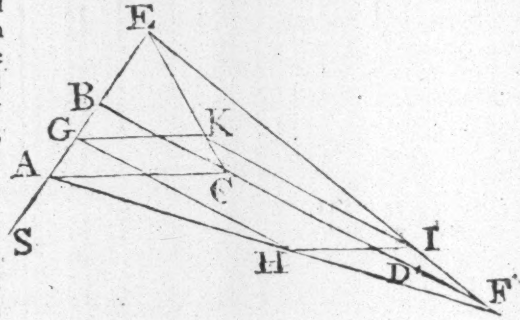
Limitation. When the circular segment described on D H neither cuts nor touches C B the question is impossible; it is also evident that whether the point D be in A C or A C produced the method will still be the same.

In the same manner it was answered by Mr. John Burrow; *also very elegant geometrical solutions were received from Messrs.* Ainsworth,

Bonnycastle, Barker, Dalby, *and* Dixon; *and algebraic answers from Messrs.* James *and* Watson.

The same answered by [the] *Rev. Mr.* Crakelt, *of* Northfleet, *in* Kent.

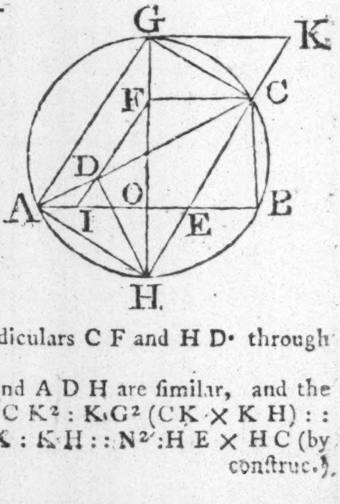
Construc. Make B L = the given perimeter, the ∠ D B S = the given vertical one, and B E and D F each equal the given difference betwixt the base and the side, and join points E and F: take E G at pleasure, and parallel to B D draw G H equal to 2 G E; also to E F apply H I = G E, and complete the parallelogram G H I K: then thro' the points E, K, draw the line E K C, and afterwards C A parallel to K G, and A B C will be the triangle required.



Demon. By similar triangles E G : G K = H I (Euc. 1. 34.) :: E A : A C. But H I = E G, by *constr.* ∴ E A = A C, and consequently the difference betwixt A C and A B is E B.------Moreover, by sim. triangles, E A : E G :: E C : E K :: C F : K I or G H; but G H = 2 G E, by *constr.* ∴ C F = 2 E A = E A + A C; and of course, since B E and D F are equal, [C F + F D or C D = A E + E B + A C or A B + A C, and adding $\overline{BC}$ to each] A B + $\overline{BC}$ + C A = B D the given perimeter. Q. E. D.

XII. Question, *answered by Mr.* George Sanderson.

Construc. Let M be half the given base, and N half the difference of the sides, make C E equal to given bisecting line on which produced, take C K to E K in the duplicate ratio of N to M; again by prop. 5. cor. 3. D. 1777. produce K E to H, so that C K : K H :: N^2 : H E × H C and having erected on K H the perpendicular C G to meet a semicircle described on K H, join G H on which describe the circle H A G B and thro' E draw A, E, B, to cut G H at right angles in O, and meet the circle in A and B join A C, B C and A C B is the triangle required.



Demonst. Join A H, A G, G K and to G H and A C draw the perpendiculars C F and H D· through D draw F I meeting A B in I.

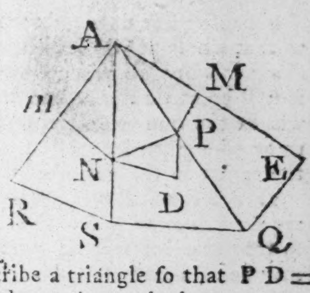
The right angled triangles G C H and A D H are similar, and the triangle K C G is similar to H C G ∴ C K^2 : K G^2 (C K × K H) :: A D^2 : A H^2 (= H E × H C) :: C K : K H :: N^2 : H E × H C (by construc.)

construc.) wherefore $AD = N$ equal (by prop. 9. page 40. Diary, 1777.) to half the difference of AC and BC. moreover as the right angles CFH and CDH are subtended by the same base CH, a circle will pass through the four points C, F, D, H, whence the angle $DFH = DCH = AGH$, (Euc. 21. 3.) wherefore FI is parallel to AG. and by sim. triangles $AD^2 : GC^2 :: AH^2 : GH^2 :: AC^2 : AG^2$ whence alternately $AD^2 : AO^2 :: GC^2 (= GF \times GH) : AG^2 (= GO \times GH) :: GF : GO :: CK :: KE :: N^2 : M^2$ (by construc.) again by alternation $AD^2 : N^2 :: AO^2 : M^2$ but $AD = N$ therefore $AO = M$ but AO equal to half AB (Euc. 3. 3.) therefore M equal to half AB. Q. E. D.

Algebraical solutions were also received from Messrs. Barker, Bowerburne, Fininley, James, Hampshire, Merritt, Pepys, Watson and Weetman, &c. *and geometrical ones from Messrs.* J. Burrow, *and* I. Dalby.

XIII. Question, *answered by Mr.* Isaac Dalby, *the proposer.*

This question requires that when the three planes are formed into a solid angle, the two first planes may make a given angle; which is a case that sometimes happens among carpenters and stone-cutters, &c. and its reverse being equally necessary, I shall here give solutions to both; as they may sometime happen to be useful to the practical workman.



First, Let RAS, SAQ, QAE be three given plane angles supposed to be spread out on a plane; it is required to find the angle that would be made by the planes RAS, EAQ supposing them revolved about SA and AQ till RA coincide with AE.

Take Am equal AM, of any given length, and draw MP and mN perpendicular to AE and AR meeting AQ and AS in P and N, and on PN describe a triangle so that $PD = PM$, and $ND = Nm$, then NDP is the angle required.

The demonst. will be evident by conceiving the triangle NDP revolved about NP when AR and AE coincide, for then the points m, D and M coincide also.

Secondly, Let the angular planes RAS, QAE be given to find SAQ so that when RA coincides with AE the angle made by those planes may be given.

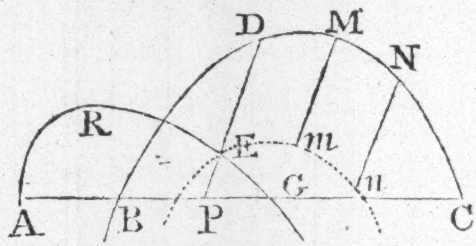
Take $Am = AM$ and draw mN and MP as before, then make an angle NDP equal the given one and take $DN = Nm$ and $DP = PM$, then on NP make a triangle whose sides are the given lines NA, AP which determines the required triangle.

Solutions were also given by Messrs. Ainsworth, Bonnycastle, Hampshire, Sanderson *and* Pepys.

XIV. Question, *answered by the Proposer.*

In the given curve BDC let there be taken as many points D, M, N, &c. as are sufficient to determine a curve of the same kind (for example, 3 for a circle, 5 for a conic section, &c.) and parallel to the line

given in position draw M m, N n, &c. equal to the given line; then through the points m, n, &c. draw a curve of the same kind as the given one, meeting A R G the other given curve in E; then D E P will be the line required: For if from a point, lines be drawn, and lines be taken in each having a constant ratio, then if the extremes of one fall in a curve of any kind, the extremes of the others will fall in curves of the same kind; and when the point is at an infinite distance, as in the present case all the lines D E, M n, will be equal, and B E m n equal to B D M N.

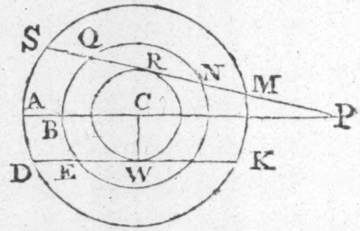


This Problem will be found of great use in resolving and determining the limits of many geometrical questions; I shall only subjoin two of the easiest examples.

1. Let there be given, two circles in magnitude and position and also the direction of a line, it is required to intercept between the peripheries of the two circles, a line parallel to this line which may be the least or greatest possible.

Suppose a line drawn through the centre of one of the circles parallel to that whose position is given; then if this circle be supposed to move along this line till it touch the other, it is evident that when it touches it will be at its limit, and in that case the distance of their centres will be the sum or difference of their radii: If the part intercepted is to be of a given length it is only requisite to set off that length from the centre in the line aforesaid, and intersect the other with the radius.

2. A S F and B S Q are two concentric circles and P a given point; required to draw a line through P, so that the intercepted part Q S may be of a given length.



Join P C and draw D E W parallel to P C so that D E may be the given length, let C W be perpendicular to D W and with the distance C W describe a circle and draw a line from P to touch it, meeting the other circles in Q and S then S Q is the given line. For D K = S M therefore D W = S R but E W = Q R consequently D E = S Q; and the limits are determined by the last.

The construction is the same whether the point be within or without the circle, and serves for all that Dr. *Horsley* has split into a dozen cases, and filled eight pages of his Book of Inclinations with; and here it may not be amiss to observe that the limits of the problems contained in that book, may be determined in a much simpler manner than that used by the Doctor; from this principle; that if the rectangle of two quantities be given, their sum will be least when their difference is least, and their sum greatest when their difference is greatest.

This question was also answered by Messrs. Ainsworth, Bonnycastle, Dalby, *and several others.*

XV. QUES-

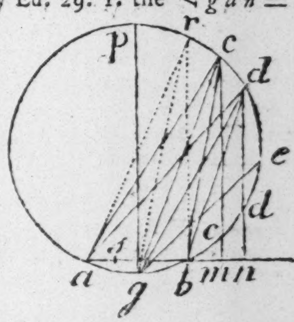
XV. Question, *anſwered by* Mr. J. H.

As the ſection of the ſolid in any part whatever, parallel to the horizon remains the ſame, it is evident the content of the ſolid will be the ſame as that of a priſm of the ſame height; and as the velocity of any point aſſumed in the triangle may be conceived to be divided into an equable horizontal motion, and a vertical one uniformly accellerated, the lengths of the ſpiral curves deſcribed by the angular points will be the ſame as thoſe of parabolic arches. *The manner of determining the ſurfaces of figures generated in a ſimilar manner will be ſhewn in a future number.*

In the ſame manner it was anſwered by Mr. Ainſworth, *and ſome others.*

Prize Question, *anſwered by* Mr. Jeremiah Ainſworth.

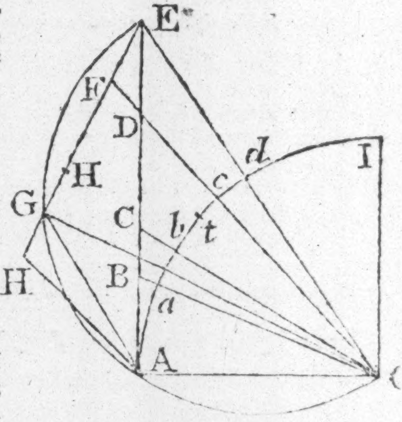
Upon any line $a\,b$ taken at pleaſure deſcribe a ſegment of a circle to contain an angle equal to the common difference of the arches; which biſect by the diameter $g\,p$, and make the angles $p\,g\,d$, $p\,g\,c$ equal to the reſpective diſtances of the middles of the equal arches from the *beginning* of the quadrant: Then if e be the middle of the ſemicirc. $g\,e\,p$, the angles $d\,g\,e$ and $c\,g\,e$ will evidently be their diſtances from the *middle*. On $a\,b$ produced * let fall the perpendiculars $d\,n$, $c\,m$ and join $d\,a$, $d\,b$, $c\,a$, $c\,b$; then ſince by Eu. 29. 1. the $\angle g\,d\,n = d\,g\,p$ and $g\,c\,m = c\,g\,p$, alſo $g\,d\,b = g\,c\,b$ $=$ by conſtruction, $\frac{1}{2}$ the common difference of the arches. It therefore follows that the 4 arches themſelves will be the meaſures of the $\angle$'s $n\,d\,b$, $n\,d\,a$; $m\,c\,b$ and $m\,c\,a$ reſpectively; conſequently the ratio of their reſpective tangents will be that of $b\,n$ to $a\,n$, and $b\,m$ to $a\,m$. Of the two arches $e\,c$, $e\,d$ let $e\,c$ be the greater, and conſequently $b\,n$ greater than $b\,m$. take $b\,n : b\,a :: b\,m : b\,s$. then ſince $b\,n$ is greater than $b\,m$, $b\,a$ is greater than $b\,s$, and conſequently $a\,m$ than $s\,m$. but by compoſition $b\,n : a\,n :: b\,m : s\,m$. and becauſe $s\,m$ is leſs than $a\,m$, $b\,m$ has to $s\,m$ a greater ratio than it has to $a\,m$; conſequently $b\,n$ has to $a\,n$ a greater ratio than $b\,m$ has to $a\,m$. Q. E. D.



And when $e\,d = e\,c$. the propoſition is evident — therefore *all* the caſes are demonſtrated. — * that the $\perp$ $c\,m$ muſt fall upon $a\,b$ produced appears from hence; that if the diameter $a\,r$ be drawn and $r\,b$ joined, it will be $\perp$ to $a\,b$ (by Eu. 31. 3.) therefore by the nature of the queſt. and conſtruction the $\angle p\,g\,c$ can never be equal to or leſs than half the common difference of the arches $= g\,r\,b = p\,g\,r$.

The ſame anſwered by Mr. George Sanderſon.

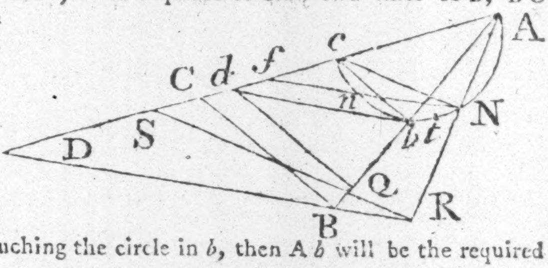
Let A I be the given quadrant, t the middle point, O the center, and A a, A b, A c, A d, the four arches taken ſuch that $a\,b$, the diff. of the two firſt may equal to $c\,d$ the diff. of the two laſt; alſo let A B, A C, A D, and A E be tangents to the ſaid arches; and having joined O B, O C, O D,



O D, O E, on O E as a diameter, describe the semicircle O A E, to which draw O G making the angle E O G = A O C. join E, G, cutting O D produced in F, and A H drawn || to O D in H. Then because A E is || to O I and A H to O D the angle E A H = F O I (= C O I) and the angle E A G = E O G (Eu. 21. 3.) = (by const.) A O C or A O b. But as the arch $a\,b$, = $c\,d$, and A t, = t I, when $b\,t$, is greater than $t\,c$, the angle A O b, (A O C) is manifestly less than F O I (c O I) and when equal, equal; and when less, the angle A O b (A O C) is greater than F O I (c O I). Whence the angle E A H is accordingly greater equal to or less than the angle E A G, and the line E H, accordingly greater equal to or less E G, but the triangles G O E and G O F are respectively equiangular to the triangles A O C and A O B ∴ A C : A B : : G E : G F and by similar triangles A E : A D : : H E : H F; but when H E is equal to G E (or the point H falls in G) H F = G F, then G E : G F : : A C : A B : : A E : A D, again when H E is greater than G E the ratio of G E to G F is greater than that of H E, to H F, and when less, less, (as is demonstrated *Barrow*'s Geom. Lectures, page 121). Therefore when H E is greater than G E ($b\,t$ greater than $c\,t$) C A to AB is a greater ratio than E A to A D, and when less, less. Q. E. D.

The same answered by Reuben Burrow.

Lemma. D A and D B are two lines given in position, and A a given point in D A; it is required to draw two lines A B, B C making A B C a given angle so that A C may be the least possible.



Take any line A c and describe a segment thereon containing the given angle, and draw $d\,b$ parallel to D B touching the circle in b, then A b will be the required position of A B.

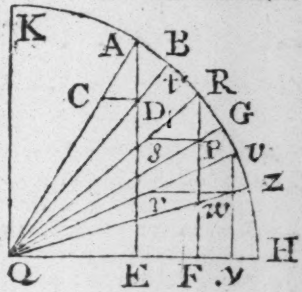
For by similar triangles d A : A c : : D A : A C, and if any other line A R be drawn, and R S make the given angle with it, and N f be parallel to B D, then f A : A c : : D A : A S but the ratio of d A to A c is greater than that of f A to A c and consequently the ratio of D A to A C is greater than that of D A to A S wherefore A S is greater than A C.

Corollary 1. Let the given angle A B C be right and d Q perpendicular to A b, then if it was required to find two arcs differing by a

given quantity ſo that the tangent of the greater ſhould have to the tangent of the leſſer the leaſt ratio poſſible, the greater arc would exceed 45° by half the given difference, and the leſſer would be leſs than 45° by half the difference: for if A t be half the ſemicircle, A c b is $= 45° + bct = 45° + \frac{1}{2}$ A d b and b d Q $=$ A d Q $-$ A d b $=$ A c b $-$ A d b $= 45° - \frac{1}{2}$ A d b $=$ D A B.

Corollary 2. Hence alſo the more the angle D A B varies from 45° $- \frac{1}{2}$ *the diff.* the ratio of D A to D C becomes the greater.

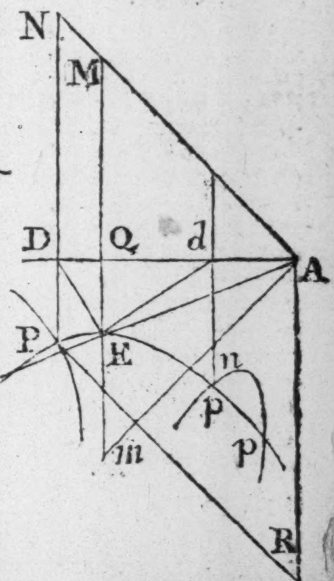
To apply this to the queſtion. Let K H be a quadrant, biſected in t, and let A H and B H, R H and G H be four arcs whoſe differences A B and R G are equal; then if A B and R G are ſo poſited that t B $= t$ R then (becauſe radius is a mean proportional between the tangent and cotangent) t. R H : r : : r : t. H B and t. G H : r : : r : t. A H wherefore t. R H : t. G H : : t. A H : t. H B that is R F : F P : : A E : E D but R F : F P : : R Q : Q S and A E : E D : : A Q : Q C, therefore A C and R S are equal; but if any arc v z equal to R G be taken at a greater diſtance from t than R G, and v w y be drawn $\perp$ to Q H meeting Q z in w, and w r be parallel to Q N meeting Q v in r, then v r will be greater than R S by the lemma, and conſequently the ratio of v Q to Q r or v y to y w, is greater than that of R F to F P, and on the contrary when v z is taken nearer to t, the ratio will be leſs.



The Prize of Twelve Diaries was the lot of Mr. Ainſworth, *and that of Six, of Mr.* George Sanderſon.

Anſwer to the PRIZE QUESTION, *omitted laſt Year.*

Lemma. Given the baſe, an angle at the baſe and the rectangle made by the ſide adjoining to it, and the ſum or difference of the other two; to find the triangle.



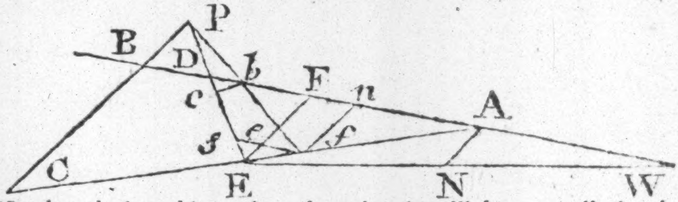
Let AE be the baſe, EAD the given angle, draw EQM perpendicular to AQ and take QM $=$ Qm $=$ QA and with the aſymptotes A M, A R, and A m, A R, deſcribe two hyperbola's whoſe powers are the rectangles of the ſum and difference reſpectively; alſo deſcribe an equilateral hyperbola whoſe axis is E Q meeting the others in P, p, and draw P D, p d parallel to A R meeting A D in D, d; then A D E, A d E are the triangles required.

For D A $=$ D N and D E $=$ D P therefore A D $+$ D E $=$ N P, and (A D $+$ D E) $\times$ A D $=$ P N $\times$ A D $=$ A N P R $=$ the given power of the hyperbola: Alſo d E $= d p$ and A d $= d n$, therefore d E $- d$ A $= n p$ and

consequently $(dE - dA) \times dA = np \times dA = nprA =$ the given quantity.

Hence if any point E be assumed in A C (see Fig. 2.) and E F be parallel to A N and F M = F E and if D be found by this lemma, so that $(AD + DE) \times DE = AM \times EW$ then if a line be drawn thro' the point P || to D E a similar triangle will be determined.

To apply this to the problem in question, Let P C be parallel to



A N, then it is evident that the triangle will have no limit when drawn on the side of P C farthest distant from A, but when drawn on the contrary side, there may be found such a triangle D E F as will when the question is possible be the limit of the triangles drawn on each side of it; which limit may be thus investigated.

Let P f be drawn indefinitely near P E, also bc parallel to A C and fes to A B, then because the triangle D E F is at its limit, by the supposition; the triangle bfn will be ultimately equal to it and their variations also equal, that is $Dc + Db + Ee = Fn$. But $DA : DE :: Db : Dc$, and $FA : FE :: Fn : Ee$ therefore $Dc + Db + Ee = \frac{DE}{DA} \times Db + Db + \frac{FE}{FA} \times Fn = Fn$, hence $Db : Fn :: (FA - FE) \times AD : (AD + DE) \times AF$; but $Db : sf :: PD : PE$ and $sf : FE :: DA : AE$, and $fE : Fn :: AE : AF$ consequently $Db : Fn :: PD \times DA : PE \times AF$; but $PD \times DA : PE \times AF :: (AF - FE) \times AD : (AD + DE) \times AF$, wherefore $PD : PE :: AF - FE : AD + DE$: Let E W be parallel to A D meeting P A in W, and $FM = FE$ then because $PD : PE :: DA : EW$ therefore $DA : EW :: AF - FE : AD + DE$ that is $AD : EW :: AM : AD + DE$. Hence this.

Construction. Draw P D E (by the foregoing lemma) so that $AD \times (AD + DE) = AM \times WE$ and E F parallel to A N, then D E F is the triangle required.

N. B. The solution of the barter question at p. 20. being founded on Malcolm's false principle, which hath generally been used by most of the modern writers, I drew up, and intended to give a paper on that subject in last year's Diary, but being obliged to defer it, shall here insert two cases only, from whence the aforesaid solution may be corrected and answers given to problems of a similar kind.

1. Suppose A has goods which he sells at a, but barters at b to have $\frac{m}{n}$ parts of the amount in ready money; B has goods worth c each; what price must he rate them at, to be equivalent to A's barter price?

Let x be the price required, then as A receives $\frac{mb}{n}$ in ready money for each piece bartered which is only worth a, the remaining part of

each

each piece which is not paid for but exchanged is only worth $a - \frac{mb}{n}$; but A values it in barter at $b - \frac{mb}{n}$, therefore c must have the same ratio to x as $a - \frac{mb}{n}$ to $b - \frac{mb}{n}$: Hence this rule.

Subtract $\frac{m}{n}$ parts of A's barter price from his selling price, and also from his barter price and say, as the first remainder is to the second, so is B's selling price to his barter price required.

2. Suppose A has goods worth a each which he charges in barter at b; and B has goods worth c each, which he would barter with A, to have $\frac{m}{n}$ parts of the whole in ready money; at what price must B rate his pieces to be equivalent to A's barter price?

Let x be the price sought, then $\frac{mx}{n}$ is the ready money B receives each piece he barters, and therefore the real value of that part of each piece which is not paid for, but exchanged, is $c - \frac{mx}{n}$; but its barter price is $x - \frac{mx}{n}$, wherefore $c - \frac{mx}{n}$ must be to $x - \frac{mx}{n}$ as a to b, and consequently by reduction $a + \frac{m}{n-m} \times b : b + \frac{m}{n-m} \times b :: c : x$ which gives this *rule*.

Add $\frac{m}{n-m}$ parts of A's barter price to his selling price, and also to his barter price, and say; as the first sum to the second, so is B's selling price, to his barter price.

A correct solution to the 12th question will be inserted next Year.

As to S. Clark's objections in the T. and C. Magazine, they can impose upon none but such as are as great dunces as himself and therefore deserve no notice.

ARTICLE XVII.

Additional Remarks on the Equation of Payments. By Reuben Burrow.

"THE *ingenious and learned professer Hutton, Esq.*" having in the last edition of his Arithmetic, introduced a new and very polite method of confuting the arguments advanced in the Diary for 1777, on the subject of equation of payments; *viz.* by representing the writer as a "*malicious defamer and an ignorant pretender*;" and notwithstanding the authority of so considerable a personage, there being still many people so obstinate as to retain their former opinion, that *abuse* is not *demonstration*,

demonſtration, and that falſe reflections on a perſon's moral character ſhould have no place in matters of ſcience; I have therefore in reſpect to ſuch of my readers, taken the liberty, of giving ſome farther confirmations of what I before advanced, and alſo to ſhew, that the rule which the "ingenious profeſſor" affirms to be "*the only* "*true one*," is not only falſe, but even falſe on his own principles; that both Kerſey's principle and Malcolm's when rightly applied bring out exactly the ſame concluſion as the old method which he has reprobated, and that the *learned profeſſor's* miſtakes ariſe from not knowing how to find the amount of a ſum of money for a given time at ſimple intereſt.

As compound intereſt

"Is an increaſe of money day by day
"And month by month, exactly in proportion
"To the elapſe of time"————

ſo ſimple intereſt is univerſally allowed to be that whoſe intereſt is ſuppoſed to bear no intereſt; or which is the ſame, it is a ſum of money payable *at the end of the time* of any tranſaction, for the uſe of money during that time, according to agreement: the truth of this will fully appear from the general practice of the beſt writers, and from common acceptation; for when tranſactions are ſettled according to ſimple intereſt, the ſuppoſition that the intereſt bears no intereſt is the ſame thing as ſuppoſing it to be of no advantage either to the debtor or creditor during the time of the tranſaction; but to have the uſe of money is certainly an advantage, and therefore the intereſt cannot be payable till the end of the time; for if it be payable ſooner, the creditor has doubtleſs a right to uſe it, and of courſe acquires an advantage by it, or which is equivalent, gains intereſt upon intereſt according to ſome ſpecies of compound intereſt; which is contrary to the ſuppoſition; and conſequently either the intereſt is not due till the end of the time, or elſe it is no advantage to have the uſe of money: But perhaps the profeſſor, as being a ſchoolmaſter, may like the argument better in this form:

1. If any thing be payable in the intermediate time, it muſt be intereſt;

2. But the intereſt is allowed to be of no uſe to either party in the intermediate time;

3. Therefore, that which is payable before the end of the time is of no uſe to either party. Now how the profeſſor will contrive to pay the intereſt ſo as to be of no uſe to either party I cannot deviſe, unleſs he do it in the new halfpence that nobody will take;—but falſe principles are not to be eſtabliſhed by a quibble.

Corollary 1. Hence if a ſum of money be put out to intereſt for a given time, the creditor has no right to half the intereſt at the expiration of half the time, but only to ſuch a part as would amount to half the intereſt at the end of the time; and ſo for other intervals, &c.

Corollary 2. Hence alſo, if 1l. be put out to intereſt at the rate r for the time t; its intereſt for any time x leſs than t is $\frac{rx}{1+rt-rx}$: For $1 : r :: t-x : rt-rx$ the intereſt of 1l. from the time x to the expiration of the remainder of the time t, therefore $1+rt-xr$ is the

x payable

amount of 1l. in that time; but rx is the interest of 1l. for the time x payable at the end of the time t, wherefore $1+rt-rx : 1 :: rx : \frac{rx}{1+rt-rx}$ the value of the interest at the end of the time x: Also the amount of 1l. in the time x when put out as above will be $\frac{1+rt}{1+rt-rx}$, which becomes $1+rt$ when x is equal to t.

This being premised; let M and N be two sums of money; the first payable directly, and the other at the end of the time t; to find the equated time according to Kersey's principle.

Let x be the equated time; then because the amount of 1l. in the time t is $1+rt$, the present value of N is $\frac{N}{1+rt}$ and therefore $M+\frac{N}{1+rt}$ or $\frac{M+N+Mrt}{1+rt}$ is the sum of the present values, which according to *Kersey* must be equal to the present value of $M+N$ payable at x; now by cor. 2. the amount of 1l. in the time x is $\frac{1+rt}{1+rt-rx}$ therefore $(M+N)\times\frac{1+rt-rx}{1+rt}$ is the present value of $M+N$ which being made equal to $\frac{M+N+Mrt}{1+rt}$ we have $x=\frac{Nt}{M+N}$ which agrees with the old method.

The same data being supposed, let Malcolm's principle, of the equality of interest and discount at the equated time, be applied: Then the interest of M for the time x being by cor. 2, equal to $\frac{Mrx}{1+rt-rx}$, and the amount of 1l. from the equated time x to the time t being $1+rt-rx$, and the interest $rt-rx$, the discount of the sum N at the time x will be $\frac{Nrt-Nrx}{1+rt-rx}$, which being made equal to $\frac{Mrx}{1+rt-rx}$ the interest; x is found equal to $\frac{Nt}{M+N}$ the same as before, and therefore both Kersey's and Malcolm's principles rightly applied agree exactly with the old method.

To prove that the professor's conclusion is false, on his own principles, nothing more is requisite than to calculate the interest from his equated time to the time of the last payment, according to his own method, and it will be found that the creditor will gain more this way, than he could by receiving the payments as they become due, which must certainly be a disadvantage to the debtor, unless the professor can demonstrate that two people may deal together upon equal terms; that one can have no advantage but what he derives from the other, yet one of them shall gain and the other shall not lose: As to the usual pretence that the business is truly settled at Malcolm's equated time, and

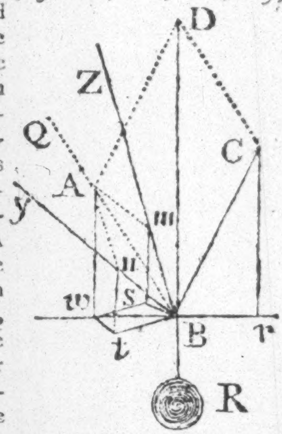
and that the time before or after it, has nothing to do in the matter; if there was any validity in it, the consequence would be, that it is best to judge of things only by halves, and that truth is variable like the ordinate of a curve, and a lie to-day may be true tomorrow, &c. &c. But it is not worth while to bestow any farther confutation on a method so grossly and palpably false, and therefore as the errors remain uncorrected in the two last editions of his book (viz. the third and fifth), I shall only advise the learned professor to correct them in his next *seventh* edition.

ARTICLE XVIII.

Miscellaneous Problems and Solutions, &c.
By Reuben Burrow.

Proposition I. Theorem.

If R be a weight supported in equilibrio by 3 cords, B C, B z, B y, knotted at B; then if a plane be supposed to pass thro' R B, B C, meeting the plane passing thro' B z, B y in B Q, the force compounded of the forces in the direction of B z, B y, will fall in B Q. For the forces in B z, B y compound a single equivalent force in the plane z B y, and if this force is not in the line B Q it may be reduced to a force in B Q and another perpendicular to it; now either the forces B A and B C in the directions B Q B C in the plane C B Q keep the body in equilibrio with respect to the direction of the plane C B Q, or not; if they do, that is, if they have equal horizontal forces, then the former force perpendicular to the plane C B Q acting at B will draw the body out of the plane and ∴ it will not be in equilibrio; if the forces B A, B C reduced to a horizontal direction are not in equilibrio, one will exceed the other, and this excess acting at B in the plane C B Q, together with the former acting at B in a direction ⊥ to this plane, will compound a third force which has no opposite force to counterbalance it, and therefore the body will not be in equilibrio, contrary to the supposition.



Proposition II. Problem.

Given the directions of three forces supporting a body in equilibrio; required the ratio of the forces.

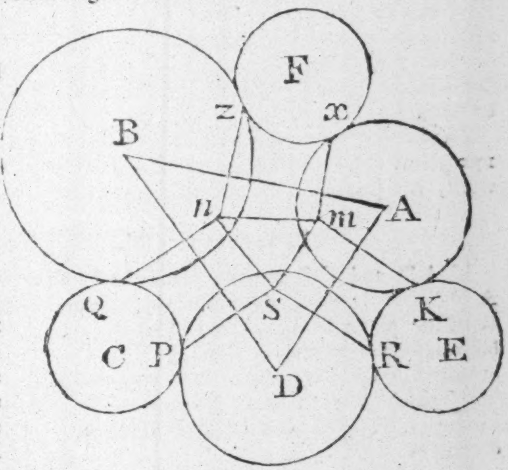
Let B C, B z, B y, be the directions of the forces and B R the direction of the force sustained; let a plane pass thro' B R, B C meeting the plane passing thro' B z, B y, in B Q; take B D = the force B R in R B produced and in this plane draw D A parallel to B C meeting the intersection of the plane B Q in A; then draw A *m* ∥ to B *y* and A *n* ∥ to B z, then B *n*, B *m*, B C express the forces of B *y*, B *z*, B P, from

from whence the practical conſtruction is evident; or the quantities may be found by calculation for $Bn = \frac{BD \times s.CBr \times cos.CBr \times s.wBt}{r \times s.tBs \times cos.nBt}$

$BC = BD \times s.CBr$, and $Bm = \frac{BD \times s.CBr \times cos.CBR \times s.wBt}{r \times s.tBs \times cos.mBs}$

Proposition III. Problem.

If three hemiſpheres be placed in a triangular ſituation, with their ſections on a horizontal plane, and a fourth ſphere be ſuſtained by the three; required the preſſure againſt each.



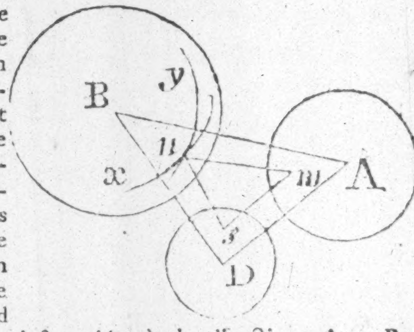
Let A, B, D be the centers of the three given hemiſpheres and E, F, C different poſitions of the fourth ſphere touching the reſt in R, K, x, z, P, Q; then if the ſphere E be ſuppoſed to revolve round the line D A, touching the two ſpheres A and D, the points of contact will deſcribe two parallel circles whoſe projections R s, K m will paſs through the points of contact R and K, and be projected into right lines on the horizontal plane; by the ſame reaſon when the ſphere revolves about D B and B A the circles deſcribed by the points of contact will be projected into right lines, and conſequently the interſections of thoſe right lines give the projections of the points of contact (made by the ſphere when ſuſtained at reſt) upon the plane of the horizon; wherefore the point m, n, s thus found are given, and alſo the lines A m, D s, B n; now if n, m, s be ſuppoſed to repreſent the points of contact made by the third ſphere when ſuſtained by the reſt, all the forces ſuſtaining it paſs through it's center in the directions A m, B n, D s; wherefore, the directions of the forces being given, the forces themſelves may be found, by the laſt problem.

Proposition IV. Problem.

Three ſpheres being placed on a horizontal plane ſuſtaining a fourth; required the force ſuſtained by each.

Let A, B, D, be the given ſpheres on the plane of the horizon, then if the fourth ſphere be ſuppoſed to touch the ſpheres B and D and to revolve round the line B D, the projection on the plane of the horizon of the circle deſcribed by the point touching the ſphere B will be an ellipſe whoſe axes and poſition are determinable from the

data;

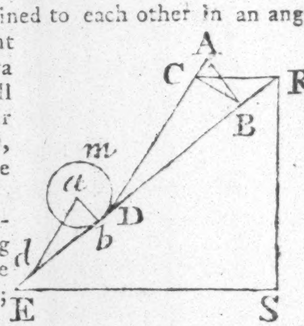


data; in the ſame manner the ellipſe deſcribed in conſequence of the revolution of the given ſphere about A B is determinable, and therefore the point of their interſection *n* may be found, and by the ſame method may the other two projections *m* and *s* of the points of contact on the plane of the horizon, be determined: Then if *n*, *m*, *s* be now ſuppoſed the real points of contact inſtead of their projections (as in the laſt problem) the directions A *m*, B *n*, D *s* will be given, and therefore the preſſure againſt each ſphere may be found by prop. 7.

N. B. The 329 queſtion of the Ladies Diary which was propoſed in 1750, but not anſwered in any of the ſucceeding publications of that work, is a particular caſe of this problem.

Proposition V. Problem.

If A D and D E are two planes inclined to each other in an angle A D E; it is required to find the point R in the plane E D produced, ſo that a body deſcending from R along R E ſhall acquire the ſame velocity at E as another body which deſcends from A down A D, impinges on D E and deſcends thence down D E to E.

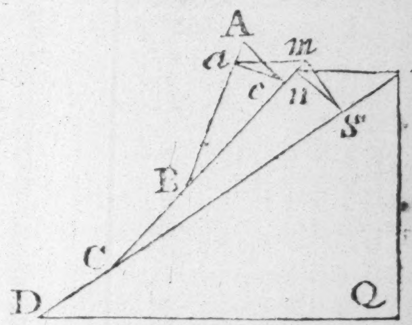


Draw A B perpendicular to E D meeting it in B, draw B C ⊥ to A D meeting it in C, and draw C R parallel to the horizontal line E F meeting E D in R; then R is the point required.

For let *a m b* be the poſition of the deſcending body (which is here ſuppoſed a globe) at the inſtant it impinges on the plane D E, and draw *a d* parallel to A D and *a b* perpendicular to D E; then becauſe any force *a d* may be reſolved into the forces *a b* and *b d*, it is evident that if *a d* repreſent the velocity acquired by deſcending through A D, the inſtant before it impinges on D E, then *b d* will repreſent the velocity with which the body proceeds along the plane D E the inſtant after the impingement, and theſe velocities are as *a d* to *d b* or A D to D B by ſimilar triangles; but A D is to D B as $\sqrt{AD}$ to $\sqrt{CD}$ and this laſt ratio is as the velocity acquired by falling through A D to the velocity acquired by falling through C D, or R D, and therefore the velocity acquired by deſcending through R D is the ſame as the body has, after deſcending through A D and impinging on D E; wherefore as their velocities are the ſame, at D, they will alſo be ſo at E, and conſequently the velocity acquired by deſcending from A to E will be the ſame as by deſcending from R to E or from R to S.

Proposition

PROPOSITION VI. PROBLEM.



If A B, B C, C D, &c. be any number of planes given in position, it is required to find the velocity that a body acquires by descending thro' them all.

Draw A n perpendicular on C B produced, and $n\ a$ ⊥ to A B, and $a\ m$ parallel to the horizon, meeting C B in m; then draw $m\ s$ perpendicular to D C produced, and $s\ c$ ⊥ to C B, and $c\ r$ parallel to D Q, meeting D C in r; then r Q is the height through which a body descending will acquire a velocity equal to that acquired by the body in falling through all the planes.

For the velocity acquired in descending from A to C is equal to that acquired by descending from m to c by the last; and by the same reason, the velocity acquired in falling from m to D is equal to that gained by falling from r to D, or from r to Q, &c.; wherefore the velocity acquired in falling through r Q is equal to that acquired in falling from A to D.

Scholium. This proposition hath been generally used by writers of mechanics, to prove that a body acquires the same velocity in descending through a curve as by falling through its perpendicular height; that this conclusion is true, appears by conceiving the above figure to become a curve, for the angles A B n, &c. being then indefinitely small, the points A, a n, m, &c. coincide, and r Q becomes the curve's altitude; but those writers deduce it from two absurd suppositions. *Maclaurin* and *James Gregory*, are, I believe, the only authors that have noticed the mistake; the first in p. 211 of his account of *Newton*'s discoveries, and the latter in a small treatise, published at *Glasgow* in 1672, under the name of *Patrick Mathers*, entitled, "*The Great and New Art of* "*Weighing Vanity.*"

New MATHEMATICAL QUESTIONS *to be answered in next Year's* DIARY.

[41] I. QUESTION, *by Mr.* George Sanderson.

IF a given chord bisect the diameter of a given circle in D; to draw a line from one end of the diameter meeting the circle in F and the chord in E so that the ratio of D E to E F may be given.

[42] II. QUESTION, *by Mr.* T. Barker.

IF two given circles be so placed that a given point P divides both their diameters in the same ratio, and S be also another given point; it is required to draw P A B cutting the two circles in A and B so that the angle A S B may be the greatest possible.

[43] III.

[43] III. QUESTION, *by Mr.* John Hampshire.

P P is a given circle and C A, C B, are lines, given in position; it is required to find the points in the circumference from whence perpendiculars being let fall on C A, C B shall make the distance of their intersections a given quantity or the least or greatest possible.

[44] IV. QUESTION, *by Mr.* J. Jackson.

IF P A B be a given triangle, and B M be drawn meeting P A in the given point M; it is required geometrically to draw P S H meeting B M in S, so that S T being drawn parallel to P A meeting B A in T, and T H being drawn parallel to P B meeting P S in H the perimeter of the triangle S T H may be either a given quantity or a *minimum*.

[45] V. QUESTION, *by Mr.* John Burrow.

IN any polygonal figure of an odd number of sides, if each two sides containing an angle be produced to meet the side opposite that angle; or if the number of sides be even, and each two sides containing an angle, be produced to meet the two sides containing the opposite angle; then will the sum of all the saliant angles of the figure thus generated be equal to two right angles, if the number of sides be odd, but equal to four right angles if the number of sides of the polygon be even.

[46] VI. QUESTION, *by Mr.* Joseph Edwards.

SUPPOSE a stick three feet long with one end resting on the palm of the hand, and inclined to the horizon in an angle of 60°; required the time of travelling 100 yards, so that the stick may preserve the same inclination to the plane.

[47] VII. QUESTION, *by Mr.* Reuben Burrow.

HAVING given two points through which a great circle is to pass; it is required to find the pole of that great circle geometrically, and also to cut off a given arch from it; according to the orthographic projection of the sphere.

[48] VIII. QUESTION, *by Mr.* Thomas Todd.

REQUIRED the curve into which a hollow cylindrical tube must be bent, so that being revolved about an axis at right angles to the horizon with a given velocity, a globe put in any part of the tube may remain there without ascending or descending.

[49] IX. QUESTION, *by the Rev. Mr.* Crakelt.

FROM a given point it is required to draw a right line cutting two circles given in magnitude and position so that the parts of the line intercepted by these circles may have a given ratio. *N. B. The* 28 *Prob.* 233 *of Simpson's Geometry is a particular case of this; but the general problem admits of a simpler construction than that there given.*

[50] X. QUESTION, *by Mr.* Thomas Moss.

TO divide a given angle into two such parts that the rectangle contained under the difference of their sines, and the sum of their cosines answering to two given unequal radii, may be of a given magnitude.

[51] XI. PRIZE QUESTION, *by Mr.* Jeremiah Ainsworth.

A Person undertakes to throw with a pair of common dice, the chance Seven before any other shall come up twice; required the exact probability of doing it?"

N. B. A solution to this Problem hath before been attempted but 'tis apprehended without success.

Whoever gives the best solution to this question before the first of May shall receive a Prize of Twelve Diaries and Companions, and the next best a prize of Six Diaries and Companions.